Universidad
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How I Learned to Stop Worrying and Love Outage Capacity

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Joint work with Wei Yang, Giuseppe Durisi, and Yury Polyanskiy

Quasi-Static MIMO Fading Channel (1)

$$\mathbf{Y}_k = \mathbb{H}\mathbf{x}_k + \mathbf{W}_k, \quad k \in \mathbb{Z}$$

- MIMO: t transmit antennas, r receive antennas.
- Signal transmitted over n channel uses.
- Dimensions: $\mathsf{X} \in \mathbb{C}^{n\times t}$, $\mathbb{Y} \in \mathbb{C}^{n\times r}$, $\mathbb{H} \in \mathbb{C}^{t\times r}$, and $\mathbb{W} \in \mathbb{C}^{n\times r}$.
- Entries of $\mathbb{W}$ are IID $\mathcal{N}_{\mathbb{C}}(0,1)$.
- Quasi-static: fading coefficients are random but stay constant.

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Capacity vs. Outage

For many interesting fading distributions channel capacity is zero:

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Outage:

$\Rightarrow$ Event that channel prohibits reliable communication at a given rate.

$\Rightarrow$ Suppose we communicate at SNR ρ, and let $H = h$ $(t = r = 1)$.
For any rate $R < \log(1 + |h|^2\rho)$ we have $\epsilon \to 0$ as $n \to \infty$.

$\Rightarrow$ Outage if $R > \log(1 + |h|^2\rho)$:

$$P_{\text{out}}(R) \triangleq \Pr\bigl(\log(1 + |H|^2\rho) < R\bigr)$$

$\Rightarrow$ Outage capacity is the supremum of all rates satisfying $P_{\text{out}}(R) \leq \epsilon$.

L.H. Ozarow, S. Shamai (Shitz), A.D. Wyner, "Information theoretic considerations for cellular mobile radio," *IEEE Transactions on Vehicular Technology*, May 1994.

E. Biglieri, J. Proakis, S. Shamai (Shitz), "Fading channels: Information-theoretic and communci-ations aspects," *IEEE Transactions on Information Theory*, October 1998.

Outage Capacity and Delay Constraints

- *"For stringent delay constraints [...] a natural information-theoretic performance measure is based on the capacity versus outage probability characteristics."*
- *"[With delay constraints], information outage probability, defined as the probability that the instantaneous mutual information of the channel is below the transmitted coding rate, is the appropriate performance limit indicator."*
- *"For practical systems with more stringent delay constraints, outage capacity is a more relevant metric."*

Is there a coding theorem for outage capacity?

$(n, M, \epsilon)_\ell$ Code

- Different scenarios: no-CSI (no), CSIT (tx), CSIR (rx), CSIRT (rt)
- An $(n, M, \epsilon)_\ell$ code ($\ell = \{\text{no}, \text{tx}, \text{rx}, \text{rt}\}$) consists of the following:

Encoder:

No-CSI or CSIR:

$f \colon \{1, \ldots, M\} \to \mathbb{C}^{n \times t}$ s.t.

$$\|f(m)\|_{\mathsf{F}}^2 \leq n\rho, \quad \forall\, m$$

Decoder:

No-CSI or CSIR:

$g \colon \mathbb{C}^{n \times t} \to \{1, \ldots, M\}$ s.t.

$$\max_{w} \Pr(g(\mathbb{Y}) \neq W | W = w) \leq \epsilon$$

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Encoder:

<u>CSIT or CSIRT:</u>

$f \colon \{1, \ldots, M\} \times \mathbb{C}^{n\times r} \to \mathbb{C}^{r\times t}$ s.t.

$$\|f(w, \mathsf{H})\|_{\mathsf{F}}^2 \leq n\rho, \quad \forall\, w, \forall\, \mathsf{H}$$

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$$\max_w \Pr(g(\mathbb{Y}, \mathbb{H}) \neq W | W = w) \leq \epsilon$$

$\Rightarrow$ maximum error probability, short-term power constraint

Maximal Achievable Rate

- Maximal achievable rate defined as

$$R_\ell^*(n,\epsilon) \triangleq \sup\left\{\frac{\log M}{n} : \exists\,(n,M,\epsilon)_\ell \text{ code}\right\}, \quad \ell = \{\text{no},\text{rx},\text{tx},\text{rt}\}$$

- What is the largest rate such that the probability of error is not larger than ϵ as $n \to \infty$?
- ϵ-capacity: $C_{\epsilon,\ell} = \lim_{n\to\infty} R_\ell^*(n,\epsilon)$

S. Verdú, T.S. Han, "A general formula for channel capacity," *IEEE Transactions on Information Theory*, July 1994.

M. Effros, A. Goldsmith, Y. Liang, "Generalizing capacity: New definitions and capacity theorems for composite channels," *IEEE Transactions on Information Theory*, July 2010.

Outage Capacity: CSIRT Case

Theorem (Caire, Taricco & Biglieri): Let $C_{\epsilon,\ell}$ be a continuous function of ϵ. Then

$$C_{\epsilon,\ell} = \lim_{n\to\infty} R_\ell^*(n,\epsilon) = \sup\{R\colon P_{\text{out,tx}}(R) \leq \epsilon\}, \quad \ell \in \{\text{tx},\text{rt}\}$$

where

$$P_{\text{out,tx}}(R) = \Pr\left(\max_{\mathsf{Q}} \log\det\left(\mathsf{I}_r + \mathbb{H}^\dagger \mathsf{Q}\mathbb{H}\right) < R\right)$$

denotes the outage probability optimized over all positive semidefinite matrices Q satisfying $\operatorname{tr}(\mathsf{Q}) \leq \rho$.

G. Caire, G. Taricco, E. Biglieri, "Optimum power control over fading channels," *IEEE Transactions on Information Theory*, July 1999.

Outage Capacity: CSIR Case

Theorem (Telatar): Let $C_{\epsilon,\ell}$ be a continuous function of ϵ. Then

$$C_{\epsilon,\ell} = \lim_{n\to\infty} R_\ell^*(n,\epsilon) = \sup\{R\colon P_{\text{out,no}}(R) \leq \epsilon\}, \quad \ell \in \{\text{rx}, \text{no}\}$$

where

$$P_{\text{out,no}}(R) = \inf_{\mathsf{Q}} \Pr\left(\log\det\left(\mathsf{I}_r + \mathbb{H}^\dagger \mathsf{Q} \mathbb{H}\right) < R\right)$$

denotes the outage probability optimized over all positive semidefinite matrices Q satisfying $\operatorname{tr}(\mathsf{Q}) \leq \rho$.

E. Telatar, "Capacity of multi-antenna Gaussian channels," *European Transactions on Telecommunications*, November 1999.

CSI at the receiver doesn't help

- For a compound channel $\{W_s \colon s \in \mathcal{S}\}$, knowledge of s at the receiver does not increase the capacity.
- Intuitively, by transmitting a training sequence of length $\propto \sqrt{n}$, the channel state can be estimated without rate loss.
- Claim follows also from our lower bounds on $R^*_{\text{no}}(n, \epsilon)$.

I. Csiszár, J. Körner, *Information Theory: Coding Theorems for Discrete Memoryless Systems*, New York: Academic, 1981.

E. Biglieri, J. Proakis, S. Shamai (Shitz), "Fading channels: Information-theoretic and communications aspects," *IEEE Transactions on Information Theory*, October 1998.

W. Yang, G. Durisi, T. Koch, Y. Polyanskiy, "Quasi-static MIMO fading channels at finite blocklength," submitted to *IEEE Transactions on Information Theory*.

Outage capacity for systems with stringent delay constraints?

Outage Capacity vs. Delay Constraints

- Outage capacity has operational meaning for $n \to \infty$.
- Quasi-static fading channel is reasonable if

$$n \ll \text{coherence time of the channel.}$$

- Suggests that

  $\Rightarrow$ delay is large.

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$$n \ll \text{coherence time of the channel.}$$

- Suggests that
  - $\Rightarrow$ delay is large.
  - $\Rightarrow$ coherence time is large.
- *"As a matter of fact, outage probability predicts surprisingly well the error probability of actual codes for practical values of n."*

G. Caire, G. Taricco, E. Biglieri, "Optimum power control over fading channels," *IEEE Transactions on Information Theory*, July 1999.

Fading Channels at Finite Blocklength

- Study how quickly $R_\ell^*(n,\epsilon) \to C_{\epsilon,\ell}$ as $n \to \infty$.
- For the Gaussian channel

$$R^*(n,\epsilon) = C - \sqrt{\frac{V}{n}}Q^{-1}(\epsilon) + \mathcal{O}\left(\frac{\log n}{n}\right)$$

$\Rightarrow$ V: channel dispersion

- What is the ϵ-dispersion of quasi-static fading channels?

Y. Polyanskiy, H.V. Poor, S. Verdú, "Channel coding rate in the finite blocklength regime," *IEEE Transactions on Information Theory*, May 2010.

MIMO Fading Channel with CSIT

Theorem: Assume that $\mathbb{H}$ satisfies the following conditions:

1. $\mathsf{E}\left[\det(\mathsf{I}_t + \rho\mathbb{H}\mathbb{H}^\dagger)\right] < \infty$.
2. The joint PDF of the ordered nonzero eigenvalues of $\mathbb{H}^\dagger\mathbb{H}$ exists and is continuously differentiable.
3. $P'_{\text{out,tx}}(C_{\epsilon,\ell}) > 0$.

Then

$$R^*_\ell(n,\epsilon) = C_{\epsilon,\ell} + \mathcal{O}\left(\frac{\log n}{n}\right), \quad \ell \in \{\text{tx}, \text{rt}\}$$

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$\Rightarrow$ ϵ-dispersion is zero!

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MIMO Fading Channel without CSIT

Theorem: Assume that the PDF of $\mathbb{H}$, denoted by $f_{\mathbb{H}}$, satisfies the following conditions:

1. $f_{\mathbb{H}}$ is smooth (has derivatives of all orders).
2. There exists a constant c such that $f_{\mathbb{H}}(\mathsf{H}) > 0$, $\|\mathsf{H}\|_{\mathsf{F}} < c$ and $f_{\mathbb{H}}(\mathsf{H}) = 0$, $\|\mathsf{H}\|_{\mathsf{F}} \geq c$.

Then

$$R_{\ell}^{*}(\epsilon, n) = C_{\epsilon,\ell} + \mathcal{O}\left(\frac{\log n}{n}\right), \quad \ell \in \{\text{no}, \text{rx}\}$$

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Stringent Conditions (?)

- MIMO case without CSIT is hard:
  - $\Rightarrow$ Q minimizing $\Pr\left(\log\det\left(\mathsf{I}_r + \mathbb{H}^\dagger \mathsf{Q} \mathbb{H}\right) < R\right)$ is unknown.
  - $\Rightarrow$ outage capacity of MIMO fading channel with no CSIT still open.
- Second condition requires that $\|\mathbb{H}\|_\mathsf{F}$ is essentially bounded:
  - $\Rightarrow$ not satisfied by common fading distributions (e.g., Rayleigh, Rician, Nakagami fading).
  - $\Rightarrow$ c can be arbitrarily large—probably a mere technicality.
- For SIMO case conditions can be weakened.

E. Telatar, "Capacity of multi-antenna Gaussian channels," *European Transactions on Telecommunications*, November 1999.

SIMO Fading Channel without CSIT

Theorem: Assume that:

1. The PDF of $\|\mathbb{H}\|_F^2$ is continuously differentiable.
2. $P'_{\text{out,no}}(C_{\epsilon,\ell}) > 0$

Then

$$R_\ell^*(n,\epsilon) = C_{\epsilon,\ell} + \mathcal{O}\left(\frac{\log n}{n}\right), \quad \ell \in \{\text{rx}, \text{no}\}$$

W. Yang, G. Durisi, T. Koch, Y. Polyanskiy, "Quasi-static SIMO fading channels at finite blocklength," *2013 IEEE International Symposium on Information Theory (ISIT)*, Istanbul, Turkey.

SIMO Fading Channel without CSIT

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$\Rightarrow$ **ϵ-dispersion is zero!**

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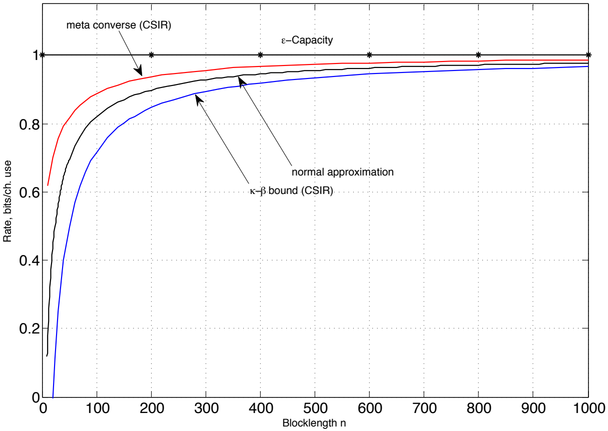
ϵ-Dispersion is zero!

$$R_\ell^*(n, \epsilon) = C_{\epsilon,\ell} + \mathcal{O}\left(\frac{\log n}{n}\right)$$

- Suggests fast convergence to outage capacity.
- Consistent with statement by Caire et al. that *"outage probability predicts surprisingly well the error probability of actual codes for practical values of n."*
- Continuity assumptions on the PDF of $\mathbb{H}$ satisfied by most common fading distributions.
- Assumptions are violated for (nonfading) Gaussian channel
  $\Rightarrow$ has in fact positive dispersion

A Numerical Example



SIMO Rician fading channel, $r = 2$, K-factor 20 dB, SNR = -1.55 dB, $\epsilon = 10^{-3}$

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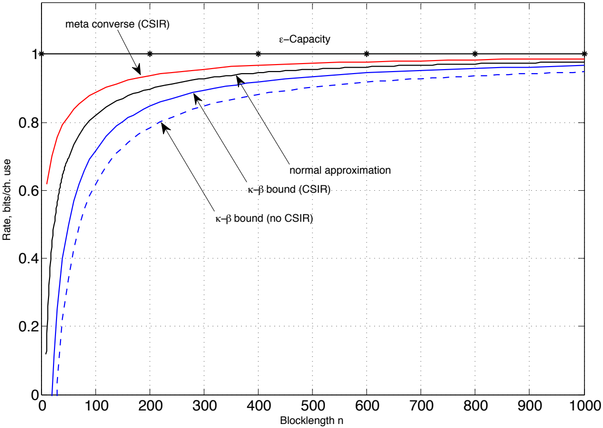

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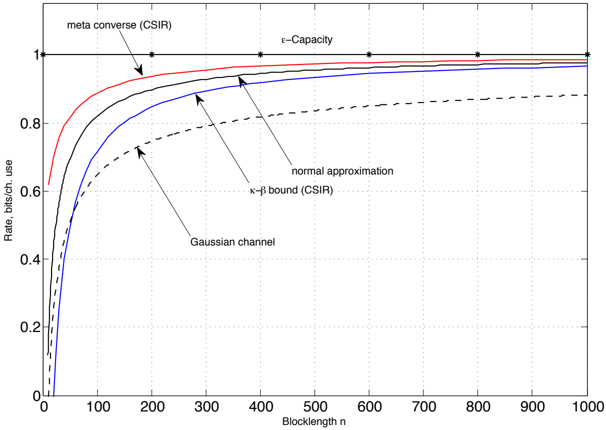

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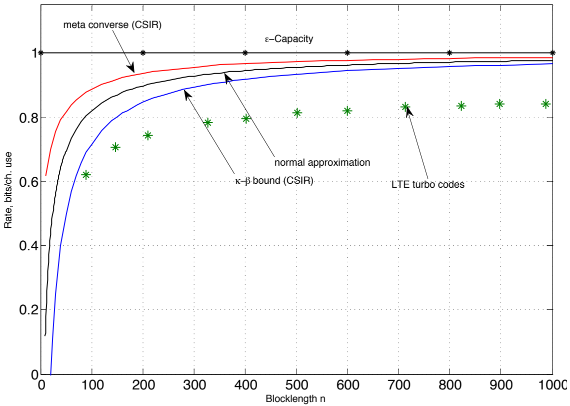
Comparison with LTE-Advanced



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Turbo codes in LTE-Advanced (QPSK, 10 iterations of max-log-MAP decoder).

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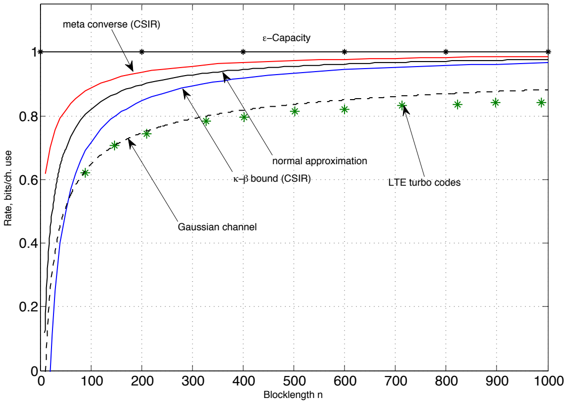

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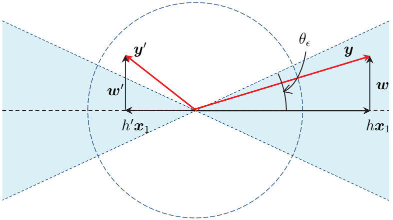
Proof Outline: Achievability (1)

- Lower bound on $R_\ell^*(n,\epsilon)$ does not require CSIR.
- Consider physically degraded channel $\mathsf{X} \to \Omega_{\mathbb{Y}} = \mathrm{span}(\mathbb{Y})$
  $\Rightarrow$ has smaller $R_\ell^*(n,\epsilon)$ than original channel $\mathsf{X} \to \mathbb{Y}$.
- CSIT: water-filling power allocation.
- Apply $\kappa\beta$ bound.
- Use as hypothesis test $Z_{\mathsf{X}}(\Omega_{\mathbb{Y}}) = \mathrm{I}\left\{\sin^2\{\mathrm{span}(\mathsf{X}), \Omega_{\mathbb{Y}}\} \le \gamma_n\right\}$
  $\Rightarrow$ $\gamma_n = e^{-C_{\epsilon,\ell} + \mathcal{O}(1/n)}$
  $\Rightarrow$ $\sin\{\mathcal{A},\mathcal{B}\}$ extends the notion of angle between vectors to complex subspaces using the concept of principal angles

Y. Polyanskiy, H.V. Poor, S. Verdú, "Channel coding rate in the finite blocklength regime," *IEEE Transactions on Information Theory*, May 2010.

Proof Outline: Achievability (2)



$$\|\boldsymbol{w}'\| \approx \|\boldsymbol{w}\| \approx \sqrt{n}$$

$$\frac{\langle \boldsymbol{x}_1, \boldsymbol{w}' \rangle}{\|\boldsymbol{x}_1\| \|\boldsymbol{w}'\|} \approx \frac{\langle \boldsymbol{x}_1, \boldsymbol{w} \rangle}{\|\boldsymbol{x}_1\| \|\boldsymbol{w}\|} \approx 0$$

Proof Outline: Converse

- Upper bound on R_ℓ^* requires CSIR.
- CSIT: transform MIMO channel into $\min\{t, r\}$ parallel channels.
- Apply meta-converse theorem.
- Use auxiliary channel whose output is circularly-symmetric, complex Gaussian with message-dependent variance.
- MIMO case without CSIT is technically much more involved.

Y. Polyanskiy, H.V. Poor, S. Verdú, "Channel coding rate in the finite blocklength regime," *IEEE Transactions on Information Theory*, May 2010.

W. Yang, G. Durisi, T. Koch, Y. Polyanskiy, "Dispersion of quasi-static MIMO fading channels via Stokes' theorem," submitted to *2014 IEEE International Symposium on Information Theory (ISIT)*.

Conclusions

- Channel dispersion of quasi-static MIMO fading channel is zero irrespective of availability of CSI.
- Suggests fast convergence to outage capacity
  $\Rightarrow$ outage capacity as metric for systems with delay constraints.
- Supports observation that outage probability describes accurately the performance over quasi-static fading channels of actual codes.

Further Literature

Dispersion of SISO stationary fading channels with CSIR:

Y. Polyanskiy, S. Verdú, "Scalar coherent fading channel: dispersion analysis," *2011 IEEE International Symposium on Information Theory (ISIT)*, St. Petersburg, Russia.

Finite-blocklength bounds for block-fading channels:

W. Yang, G. Durisi, T. Koch, Y. Polyanskiy, "Diversity versus channel knowledge at finite blocklength," *2012 IEEE Information Theory Workshop (ITW)*, Lausanne, Switzerland.

Second-order coding rate for quasi-static MIMO Rayleigh-fading channels when $\{t, r\} \propto n$:

J. Hoydis, R. Couillet, P. Piantanida, "The second-order coding rate of the MIMO Rayleigh block-fading channel," submitted to *IEEE Transactions on Information Theory*.

Quasi-static fading channel with long-term power constraints:

W. Yang, G. Caire, G. Durisi, Y. Polyanksiy, "Finite blocklength channel coding rate under a long-term power constraint," submitted to *2014 IEEE International Symposium on Information Theory (ISIT)*.